Negative differential conductance induced by electronic correlation in a double quantum dot molecule

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Electron tunneling through a two-stage Kondo system constituted by a double quantum dot molecule side coupled to a quantum wire under the effect of a finite external potential is studied. We found that *I-V* characteristic shows a negative differential conductance region induced by the electronic correlation. This phenomenon is a consequence of the properties of the two-stage Kondo regime under the effect of an external applied potential that takes the system out of equilibrium. The problem is solved using the mean-field finite-*U* slave-boson formalism.

PACS number(s): 73.21.La, 73.63.Kv, 72.10.Fk, 85.35.Be

I. INTRODUCTION

Many devices exhibit negative differential conductance (NDC) as multiple quantum wells, double barrier, double quantum dots, etc.[1–6] The NDC has applications as amplifiers and oscillators in the microwave, millimeter-wave, and terahertz frequency ranges. Extensive experimental and theoretical investigation have been devoted to the study of the *I-V* characteristics and NDC phenomenon in double quantum dot molecules.[5–9] Moreover, there is a wide literature on transport through double quantum dots (DQD) in different geometries, e.g., DQD in series and in parallel.[9,10] Two aspects of electronic transport through quantum dots have attracted great attention in the last years: the Coulomb blockade effect and the Kondo effect.[11–13] Recently Kondo effect has been studied in side attached[14] and parallel quantum dots.[15,16] Electron transport experiments showed that Kondo and Fano resonances occur simultaneously.[17] Multiple scattering of traveling electronic waves on a localized magnetic state are crucial for the formation of both resonances. The condition for the Fano resonance is the existence of two scattering channels: a discrete level and a broad continuum band.[18]

An alternative configuration consists of a double quantum dot molecule side coupled to a perfect quantum wire (QW).[19,20] This structure is reminiscent of the T-shaped quantum wave guides.[21] In this case, the QDs act as scattering centers in close analogy with the traditional Kondo effect.[22]

Although the electron-electron interaction does play an important role in many systems that exhibit negative differential conductance, for instance, producing bistabilities in the current, it is not the driving force of the negative conductance itself. In this work instead, we study a system in the Kondo regime with an *I-V* characteristics that possesses a NDC region that is induced by the electronic correlation itself, tuned by the external potential and by the parameters of the system.

The system is constituted by a quantum dot molecule side coupled to a quantum wire, as shown in Fig. 1. We use the finite-*U* slave-boson mean-field approach (SBMFA), which was initially developed by Kotliar and Ruckenstein[23] and

used later by Dong and Lei[24] to study the transport through coupled double quantum dots connected to leads. This approach enforces the correspondence between the impurity fermions and the auxiliary bosons to a mean-field level to release the $U=\infty$ restriction. This allows us to treat nonperturbatively the dot-lead coupling for an arbitrary strength of the Coulomb interaction U.[24] As it is well known, the finite U slave-boson mean-field formalism does not possess the pathologies of the infinite U one which produces a decoupling of the impurity site from the rest of the system above the Kondo temperature and also well inside the Kondo regime. More importantly for our case, we require a finite U treatment because otherwise the exchange interaction, one of the energies controlling the physics of the system we study, is reduced to zero.

In a previous work we study this system in a thermodynamic equilibrium situation. We found that in the weak-interaction regime, when the direct antiferromagnetic interaction between the dots is less than the Kondo temperature associated to the internal dot, the transmission spectrum shows a structure with two antiresonances localized at the renormalized molecular energies of the double quantum dot.[19] The local density of states (LDOS) of the system shows that when the Kondo correlations are dominant the system is in a two-stage Kondo regime with two different temperatures T_{k1} and T_{k2}, each one associated to a dot.

In the present paper we study this system under the effect of a finite external field, which takes it out of the thermodynamic equilibrium, modifying the Kondo regime and even destroying it for enough large fields. This process has fundamental consequences on the transport properties of the sys-
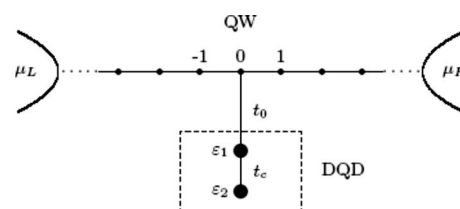

FIG. 1. Scheme of DQD attached to a lead (perfect QW). The QW is coupled to the left (*L*) and right (*R*).

tem and, in particular, creates a remarkable NDC in the *I-V* characteristics. This NDC phenomenon can be understood realizing that the applied bias destroys the lower-temperature Kondo state of the external dot modifying the capability of the other to interfere on the current that goes along the system. The scenario created by the small external bias necessary to disrupt this Kondo regime can be adequately described by SBMFA.[7]

II. MODEL

Let us consider a quantum dot molecule side coupled to a perfect quantum wire (QW) (see Fig. 1). We describe it by the two-impurity Anderson Hamiltonian. Each dot has a single level energy ε_l (with $l=1,2$) and equal intradot Coulomb repulsion U. The side attached quantum dot molecule is coupled to the QW with coupling t_0. The QW sites have zero local energies and a hopping parameter t.

The corresponding model Hamiltonian is

$$H_0 = -t \sum_{i,\sigma} (c_{i,\sigma}^\dagger c_{i+1,\sigma} + \text{H.c.}) - \sum_\sigma [(t_0 c_{0,\sigma}^\dagger + t_c f_{2,\sigma}^\dagger) f_{1,\sigma} + \text{H.c.}]$$
$$+ \sum_{l=1,2,\sigma} \left[\left(\varepsilon_l + \frac{U}{2} \hat{n}_{l,-\sigma} \right) \hat{n}_{l,\sigma} \right], \tag{1}$$

where $c_{i,\sigma}^\dagger$ ($c_{i,\sigma}$) is the creation (annihilation) operator of an electron with spin σ at the ith site of the quantum wire; $f_{l,\sigma}^\dagger$ ($f_{l,\sigma}$) is the creation (annihilation) operator of an electron with spin σ in the lth QD, $\hat{n}_{l,\sigma}$ is the corresponding number operator, t_c is the hopping matrix element between the dots, and ε_l correspond to the energy of the local states at the dots.

III. SLAVE-BOSON MEAN-FIELD THEORY

To find the solution of this correlated fermions system for finite U that guaranties that the exchange interaction between the dots is correctly treated without incorporating extra term in the Hamiltonian. We appeal to an analytical approach where, generalizing the infinite-U slave-boson approximation[25] the Hilbert space is enlarged at each site, to contain in addition to the original fermions a set of four bosons[23] represented by the creation (annihilation) operators $\hat{e}_l^\dagger$ ($\hat{e}_l$), $\hat{p}_{l,\sigma}^\dagger$ ($\hat{p}_{l,\sigma}$), and $\hat{d}_l^\dagger$ ($\hat{d}_l$) for the lth dot. They act as projectors onto empty single occupied (with spins up and down) and doubly occupied electron states, respectively. Then, each creation (annihilation) operator of an electron with spin σ in the lth QD is substituted by $f_{l,\sigma}^\dagger Z_{l,\sigma}^\dagger$ ($Z_{l,\sigma} f_{l,\sigma}$), where

$$Z_{l,\sigma} = (1 - \hat{d}_l^\dagger \hat{d}_l - \hat{p}_{l,\sigma}^\dagger \hat{p}_{l,\sigma})^{-1/2} (\hat{e}_l^\dagger \hat{p}_{l,\sigma} + \hat{p}_{l,-\sigma}^\dagger \hat{d}_l)$$
$$\times (1 - \hat{e}_l^\dagger \hat{e}_l - \hat{p}_{l,-\sigma}^\dagger \hat{p}_{l,-\sigma})^{-1/2}. \tag{2}$$

As the problem is solved adopting the U-finite SBMFA, the operator is chosen to reproduce the correct $U \rightarrow 0$ limit in the mean-field approximation without changing neither the eigenvalues nor the eigenvector.[24]

The constraint, i.e., the completeness relation $\sum_\sigma \hat{p}_{l,\sigma}^\dagger \hat{p}_{l,\sigma} + \hat{e}_l^\dagger \hat{e}_l + \hat{d}_l^\dagger \hat{d}_l = 1$ and the condition among fermions and bosons

$n_{l,\sigma} - \hat{p}_{l,\sigma}^\dagger \hat{p}_{l,\sigma} - \hat{d}_l^\dagger \hat{d}_l = 0$, is incorporated with Lagrange multipliers $\lambda_l^{(1)}$ and $\lambda_{l,\sigma}^{(2)}$ into the Hamiltonian. Also in the mean-field approximation all the boson operators are replaced by their expectation values $p_{l,\sigma}$, e_l and d_l which can be chosen, without loss of generality, as real numbers. The Hamiltonian in this new and enlarged Hilbert space is $H = H_b + H_e$, where

$$H_b = \sum_{l=1,2} \lambda_l^{(1)} (p_{l,\uparrow}^2 + p_{l,\downarrow}^2 + e_l^2 + d_l^2 - 1) - \sum_{l=1,2,\sigma} \lambda_{l,\sigma}^{(2)} (p_{l,\sigma}^2 + d_l^2)$$
$$+ U \sum_{l=1,2} d_l^2 \tag{3}$$

depends explicitly only on the boson expectation values ($e_l = \langle e_l \rangle = \langle e_l^\dagger \rangle$ and equivalently for the others operators) and the Lagrange multipliers. The Hamiltonian H_e can be written as

$$H_e = -t \sum_{i,\sigma} (c_{i,\sigma}^\dagger c_{i+1,\sigma} + \text{H.c.}) + \sum_{l=1,2,\sigma} \tilde{\varepsilon}_{l,\sigma} n_{l,\sigma}$$
$$- \sum_\sigma [\tilde{t}_{0,\sigma} (c_{0,\sigma}^\dagger f_{1,\sigma} + \text{H.c.}) + \tilde{t}_{c,\sigma} (f_{1,\sigma}^\dagger f_{2,\sigma} + \text{H.c.})]. \tag{4}$$

The tight-binding Hamiltonian depends implicitly on the boson expectation values through the parameters $\tilde{\varepsilon}_{l,\sigma} = \varepsilon_{l,\sigma} + \lambda_{l,\sigma}^{(2)}$, $\tilde{t}_{0,\sigma} = t_0 \tilde{Z}_{l,\sigma}$, $\tilde{t}_{c,\sigma} = t_c \tilde{Z}_{1,\sigma} \tilde{Z}_{2,\sigma}$, where $\tilde{Z}_{l,\sigma}$ is the value assumed by the operator $Z_{l,\sigma}$ when the four boson operators are substituted by their mean values in Eq. (2),

$$\tilde{Z}_{l,\sigma} = \frac{p_{l,\sigma}(e_l + d_l)}{\sqrt{(1 - d_l^2 - p_{l,\sigma}^2)(1 - e_l^2 - p_{l,-\sigma}^2)}}. \tag{5}$$

The boson operator expectation values and the Lagrange multipliers are determined by minimizing the energy $\langle \mathcal{H} \rangle$ with respect to these quantities. It is obtained in this way a set of nonlinear equations for each quantum dot, relating the expectation values of the four bosonic operators, the three Lagrange multipliers, and the electronic expectation values,

$$p_{l,\sigma}^2 = \langle \hat{n}_{l,\sigma} \rangle - d_l^2, \tag{6a}$$

$$e_l^2 = 1 - \sum_\sigma \langle \hat{n}_{l,\sigma} \rangle + d_l^2, \tag{6b}$$

$$\lambda_l^{(1)} = \frac{t_0}{e_l} \sum_\sigma \langle f_{l,\sigma}^\dagger c_{0,\sigma} \rangle \frac{\partial \tilde{Z}_{l,\sigma}}{\partial e_l}, \tag{6c}$$

$$\lambda_l^{(1)} - \lambda_{l,\sigma}^{(2)} = \frac{t_0}{p_{l,\sigma}} \sum_{\sigma'} \langle f_{l,\sigma'}^\dagger c_{0,\sigma'} \rangle \frac{\partial \tilde{Z}_{l,\sigma'}}{\partial p_{l,\sigma}}, \tag{6d}$$

$$U + \lambda_l^{(1)} - \sum_\sigma \lambda_{l,\sigma}^{(2)} = \frac{t_0}{d_l} \sum_\sigma \langle f_{l,\sigma}^\dagger c_{0,\sigma} \rangle \frac{\partial \tilde{Z}_{l,\sigma}}{\partial d_l}, \tag{6e}$$

where in the absence of external magnetic field the solutions are spin independent.

To obtain the electronic expectation values $\langle \cdots \rangle$, the Hamiltonian H_e is diagonalized. Their stationary states can be written as

$$|\psi_k\rangle = \sum_{j=-\infty}^{\infty} a_j^k |j\rangle + \sum_{l=1}^{2} b_l^k |l\rangle, \tag{7}$$

where a_j^k and b_l^k are the probability amplitudes to find the electron at the site j and at the lth QD, respectively, with energy $\omega = -2t \cos k$. As we study the paramagnetic case the spin index is neglected.

The amplitudes a_j^k and b_l^k obey the following linear difference equations:

$$\omega a_j^k = -t(a_{j+1}^k + a_{j-1}^k), \quad j \neq 0, \tag{8a}$$

$$\omega a_0^k = -t(a_1^k + a_{-1}^k) - \tilde{t}_0 b_1^k, \tag{8b}$$

$$(\omega - \tilde{\varepsilon}_1) b_1^k = -\tilde{t}_0 a_0^k - \tilde{t}_c b_2^k, \tag{8c}$$

$$(\omega - \tilde{\varepsilon}_2) b_2^k = -\tilde{t}_c b_1^k. \tag{8d}$$

In order to study the solutions of Eqs. (8a)–(8d), we assume that the electrons are described by incident, reflected, and transmitted plane waves with unitary, r, and τ amplitudes, respectively.[26] That is,

$$a_j^k = e^{ik \cdot j} + r e^{-ik \cdot j} \quad (k \cdot j < 0), \tag{9a}$$

$$a_j^k = \tau e^{ik' \cdot j} \quad (k \cdot j > 0). \tag{9b}$$

Inserting Eqs. (9a) and (9b) into Eqs. (8a)–(8d), we get an inhomogeneous system of linear equations for τ, r, a_j^k, and b_l^k, leading to the following expression in equilibrium ($k = k'$):

$$\tau = \frac{(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+)}{(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+) + i(\omega - \tilde{\varepsilon}_{d2})\tilde{\Gamma}}, \tag{10}$$

where the bonding ($\tilde{\varepsilon}_-$) and antibonding energies ($\tilde{\varepsilon}_+$) are defined by $\tilde{\varepsilon}_\pm = (\tilde{\varepsilon}_{d1} + \tilde{\varepsilon}_{d2})/2 \pm \sqrt{(\tilde{\varepsilon}_{d1} - \tilde{\varepsilon}_{d2})^2/4 + \tilde{t}_c^2}$ and $\tilde{\Gamma} = \pi \tilde{t}_0^2 \rho_0$ is the renormalized coupling between the double quantum dot and the quantum wire and ρ_0 is the density of states of the leads at the Fermi level. In spite of the apparent simplicity of the expression, it is necessary to remember that the quantities $\tilde{t}_0$ and $\tilde{t}_c$ implicitly depend on the expectation values of the boson and fermion operators.

The transmission probability is given by $T = |\tau|^2$,

$$T(\omega) = \frac{[(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+)]^2}{[(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+)]^2 + [(\omega - \tilde{\varepsilon}_{d2})\tilde{\Gamma}]^2}. \tag{11}$$

From the amplitudes b_1^k and b_2^k we obtain the LDOS at the quantum dot l (with $l = 1, 2$),

$$\rho_1 = \frac{1}{\pi} \frac{\tilde{\Gamma}(\omega - \tilde{\varepsilon}_{d2})^2}{[(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+)]^2 + [(\omega - \tilde{\varepsilon}_{d2})\tilde{\Gamma}]^2}, \tag{12}$$

$$\rho_2 = \frac{1}{\pi} \frac{\tilde{\Gamma}\tilde{t}_c^2}{[(\omega - \tilde{\varepsilon}_-)(\omega - \tilde{\varepsilon}_+)]^2 + [(\omega - \tilde{\varepsilon}_{d2})\tilde{\Gamma}]^2}. \tag{13}$$

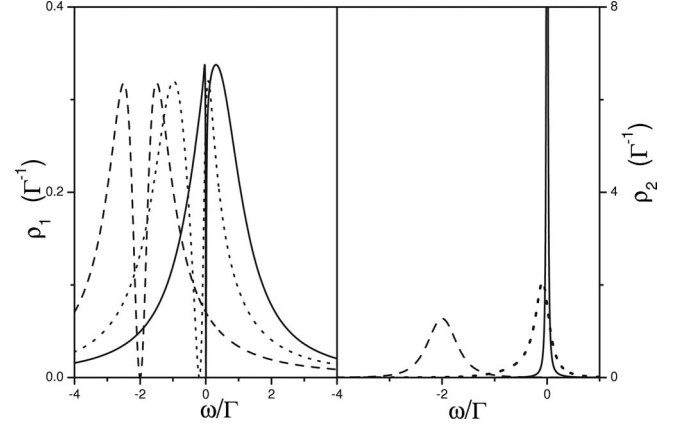In the nonequilibrium case, we suppose a finite source-drain biased with a symmetric voltage drop. Here we are

FIG. 2. LDOS at each dot (left panel, QD1 and right panel, QD2) for $t_c = 0.5\Gamma$, $V_g = -2\Gamma$ for various values of on site energy, $U = 6\Gamma$ (solid line), $U = 2\Gamma$(dotted line), and $U = 0$ (dashed line).

supposing a small applied potential, which is required to disrupt the Kondo regime controlled by the lower Kondo temperature. This fact permits us to suppose that as we are not very far from equilibrium the description provided by the approximation would be capable of properly describing the physics of the system, at least from a qualitative point of view.[7] The incident electrons from the left side (L) are in equilibrium with the thermodynamical potential $\mu_L = \mu + V/2$ and those from the right side (R) with $\mu_R = \mu - V/2$.

Once the amplitudes $a_{j,\sigma}^k$ and $b_{j,\sigma}^k$ are known, the electronic expectation values are obtained from

$$\langle f_l^\dagger c_j \rangle = \frac{1}{2} \sum_{\alpha=L,R} \frac{1}{N} \sum_{k_\alpha} f(\epsilon_{k_\alpha} - \mu_\alpha) b_l^{k_\alpha *} a_j^{k_\alpha}, \tag{14}$$

where $\epsilon_{k_\alpha} = -2t \cos k_\alpha$. The current is given by

$$I = 2\frac{2e}{\hbar} t \sum_{\alpha, k_\alpha} f(\epsilon_{k_\alpha} - \mu_\alpha) \text{Im}\{a_0^{k_\alpha} a_1^{k_\alpha}\}, \tag{15}$$

where $f(\epsilon_{k_\alpha} - \mu_\alpha)$ is the Fermi function for incident electrons from the α side and the sum on k_α is taken up to the maximum value $\cos^{-1}(-\mu_\alpha/2t)$. The quantities ε_l, the energies of the local states at the dots, are taken to be equal to $\varepsilon_1 = \varepsilon_2 = V_g$, where V_g is the gate voltage applied to the quantum dots.

IV. RESULTS

A. Thermodynamics equilibrium case

In order to obtain a more clear insight, we study first the system in thermodynamical equilibrium. The DOS at each dot of the quantum molecule is shown in Fig. 2, for various values of U, for the case $t_c = 0.5\Gamma$ and $V_g = -2\Gamma$. As U increases, $U > 2\Gamma$, the system passes from the intermediate-valence regime into a Kondo regime. This process is clearly seen to occur for QD2 where the resonance shifts and becomes sharper as the system enters into the Kondo regime. The same process takes place for the QD1 although the sharpening is less accentuated. It is noticeable that as QD2
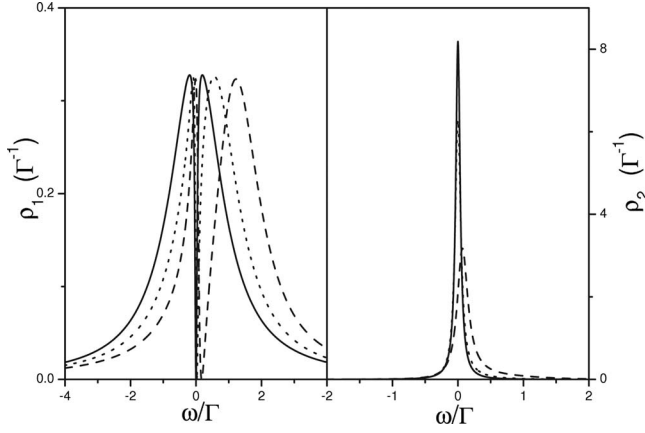
FIG. 3. LDOS at each dot (left panel, QD1 and right panel, QD2) for $t_c=0.5\Gamma$, $U=4\Gamma$ for various values of the gate voltage,$V_g=-2\Gamma$ (solid line),$V_g=-\Gamma$ (dotted line), and $V_g=0$ (dashed line).

FIG. 4. *I-V* characteristics for $t_c=0.5\Gamma$ (solid line) and $t_c=\Gamma$ (dashed line) for various values of U with $\varepsilon_1=\varepsilon_2=-U/2$.

develops a Kondo resonance, a dip appears at the Fermi energy in the LDOS of QD1. This is produced because the spin of the local electron at QD2 is Kondo correlated with the conduction-band spins through the mediation of the intermediate dot as well at the Kondo regime. This coupling creates a sharp peak at the LDOS of QD2 with width T_{k2} and a depletion of this same width at the Fermi level of the Kondo peak of width T_{k1} corresponding to QD1. This result implies the existence of a two-stage Kondo regime that appears in this system in the weak interacting limit, when $t_c^2/U<T_{k1}$,[19] being T_{k1} and T_{k2} the Kondo temperatures associated to each dot. It is the depletion of the LDOS of QD1 at the Fermi level that permits the transmission to be unity. In this case, although the side attached QD1 is in the Kondo regime, it does not provide an alternative path for the conducting electrons. As a consequence, there is no destructive interference because there is only the direct path available for them.

In Fig. 3 we show the LDOS of each QD for different values of V_g in the two-stage Kondo regime. As expected, the peaks are pinned at the Fermi energy independent of the values of V_g, a clear signal of the Kondo regime. Here it is clearly seen at the Fermi energy the antiresonance at it QD1 and the concomitant resonance at the QD2.

B. Out of thermodynamical equilibrium situation

The previous discussion was restricted to the thermodynamical equilibrium situation and was presented with the purpose of clarifying the concepts involved. When an external potential is applied the scenario changes completely. The physics of this new situation can be explained analyzing the current and the differential conductance dI/dV, two significant and experimentally measurable quantities, as a function of the applied field.

Figure 4 displays the *I-V* characteristics for two values of t_c and different values of U with $V_g=-U/2$. For $U=0$, the *I-V* characteristics shows a plateau that is related to the Fano antiresonances in the transmission spectrum. When the applied potential is of the order of the interdot interaction, the
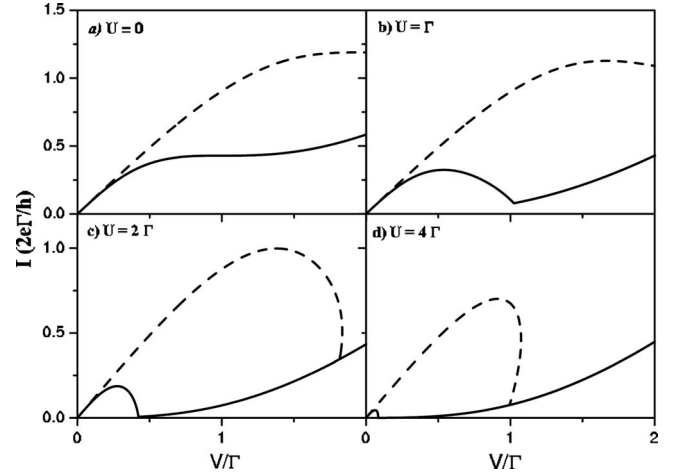
transmission, due to the Fano destructive interference, is almost zero in the bonding and antibonding regions giving no additional contribution to the total current as the applied potential is increased. This is the origin of the plateau behavior, shown in Fig. 4, when $U=0$. As U is increased, a negative differential conductance appears in the *I-V* characteristics that get more important as t_c is augmented.

Figure 5 depicts the differential conductance for the same parameters of Fig. 4. For $U=0$ it reflects essentially the transmission spectrum. As U is increased the differential conductance becomes negative in a region of the applied potential, reflecting the fact that the NDC is a consequence of the Coulomb interaction. It is a small effect in the fluctuating-valence regime $\Gamma>U$ and develops completely in the Kondo regime $\Gamma<U$, increasing with t_c.

In order to get insight into these results it is convenient to write the transmission [Eq. (11)] as the superposition of Fano and Breit-Wigner line shapes, a good approximate expression for large values of U.[19] The results is
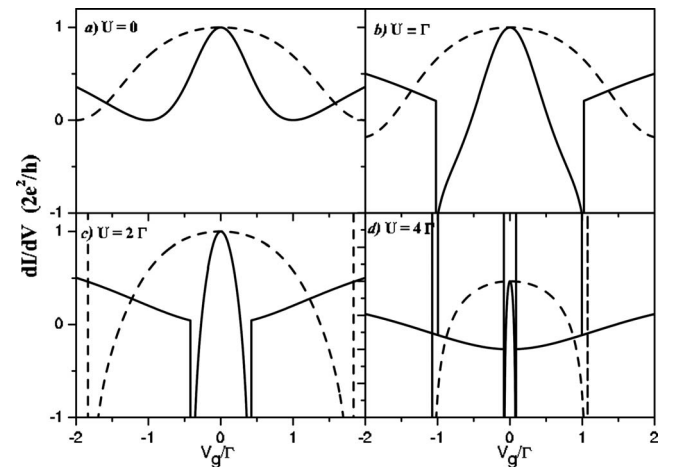

FIG. 5. Differential conductance for $t_c=0.5\Gamma$ (solid line) and $t_c=\Gamma$ (dashed line) for various values of U with $V_g=-U/2$.
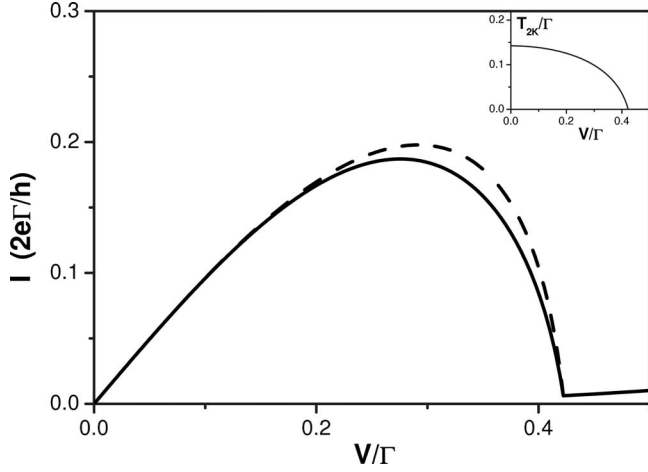
FIG. 6. Comparison of *I-V* characteristics between the numerical calculation (solid line) and the approximation [Eq. (4)] (dashed line) for $U=2\Gamma$ and $t_c=0.5\Gamma$ with $V_g=-U/2$. $T_{2K}=\widetilde{\Delta}$ in the inset.

$$T(\omega) \approx \frac{\epsilon^2}{\epsilon^2+1} + \frac{\widetilde{\Delta}^2}{\omega^2+\widetilde{\Delta}^2}, \qquad (16)$$

where $\epsilon=\omega/\widetilde{\Gamma}$ and $\widetilde{\Delta}=\widetilde{t}_c^2/\widetilde{\Gamma}$ corresponds to the two Kondo temperatures $T_{k1}=\widetilde{\Gamma}$ and $T_{k2}=\widetilde{\Delta}$.

An analytical expression can be obtained for the current by integrating over ω the transmission probability given in Eq. (16).

$$I \approx \frac{2e}{h}\left[eV - 2\widetilde{\Gamma}\arctan\left(\frac{eV}{2\Gamma}\right) + 2\widetilde{\Delta}\arctan\left(\frac{eV}{2\widetilde{\Delta}}\right)\right]. \qquad (17)$$

We identify each term of Eq. (17) as follows. The first term on the right-hand side is the contribution arising from an ideal one dimensional conductor. The second term arises from the Kondo-Fano state with temperature T_{k1} giving rise to a quasiplateau for the current and almost zero differential conductance when $|V| \ll \widetilde{\Gamma}$. The third term arises from the Kondo state weakly coupled to the wire and it is responsible for the rapid increase in the current in the region of small applied potentials.

It is important to emphasize that in this expression the quantities $\widetilde{\Delta}$ and $\widetilde{\Gamma}$ are functions of V obtained through the self-consistent calculation presented above. The case of $\widetilde{\Delta}=T_{k2}$ is shown in the inset of Fig. 6. It is clear that increasing V, both the Kondo temperature of the external dot and the current reduce to zero simultaneously with the disappearance of the NDC phenomenon. As mentioned above, this behavior can be understood realizing that the effect of the external quantum dot is to reduce the intermediate dot interference effect on the current circulating along the leads. This role is exercised by the external quantum dot as far as it is at the Kondo regime. When the external potential is large enough as to disrupt its Kondo ground state $(T_{k2} < V)$ the interference is reestablished and the current goes to zero. As T_{k2} increases with t_c, for larger values of t_c this disrupting process requires

bigger values of the applied potential as shown in Fig. 4. When $|V|$ increases still further it destroys as well the Kondo regime of the intermediate quantum dot and the current rises as depicted in this same figure. This seems to be the behavior of a two-stage Kondo system under the effect of an external potential. The Kondo regime of the outside quantum dot, not directly connected to the continuum, depends on the Kondo effect of the intermediate quantum dot. As its Kondo temperature is lower, its Kondo ground state is disrupted by lower values of V than the other dot. This process manifests in the transport properties by the appearance of a NDC region in the current. Figure 6 displays a comparison between the *I-V* characteristics of the numerical calculation and the approximation [Eq. (17)] for $U=2\Gamma$ and $t_c=0.5\Gamma$. The approximation over estimates the peak of the current; however, qualitatively, it maintains the form.

Deriving the current in Eq. (17) we obtain

$$\frac{\partial I}{\partial V} \approx \frac{2e^2}{h}\left\{\frac{\left(\frac{eV}{2\widetilde{\Gamma}}\right)^2}{1+\left(\frac{eV}{2\widetilde{\Gamma}}\right)^2} + \frac{1}{1+\left(\frac{eV}{2\widetilde{\Delta}}\right)^2}\right.$$
$$\left. + 2\frac{\partial\widetilde{\Delta}}{\partial V}\left[\arctan\left(\frac{eV}{2\widetilde{\Delta}}\right) - \frac{\left(\frac{eV}{2\widetilde{\Delta}}\right)}{1+\left(\frac{eV}{2\widetilde{\Delta}}\right)^2}\right]\right\}. \qquad (18)$$

In the above equation, the last term is responsible for the negative differential conductance. This term is negative because $\partial\widetilde{\Delta}/\partial V < 0$. This expression is able to reproduce very well the results of the differential conductance shown in Fig. 5.

Regarding the observability of the NDC in the side-coupled double quantum dot molecule, we consider the value of Γ given by Sato *et al.*,[17] $\Gamma=3$ meV. For $t_c=0.5\Gamma$ and $U=\Gamma$ the maximum and minimum currents are $I_{max} \sim 24$ nA and $I_{min} \sim 6$ nA, respectively, giving a peak to valley ratio of 4:1. The lower Kondo temperature of the external dot for these same parameters is of the order of 2 K. These values of the current and temperature are well above the experimental limits of present day techniques.

V. SUMMARY

In summary we have studied the nonequilibrium transport through a double quantum dot molecule side coupled to a quantum wire using the finite-U slave-boson mean-field approach at $T=0$ as a function of the parameters that define the system. We find that the *I-V* characteristics show a remarkable NDC, different from case reported in the literature, which is induced by the electronic correlation. This NDC behavior is a consequence of the properties of a two-stage Kondo system under the effect of an external applied potential that takes the system out of equilibrium when the applied potential is large enough to destroy the Kondo regime characterized by the lower Kondo temperature.

ACKNOWLEDGMENTS

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